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$\|\mathbf{e}_1(t)\| = 1$, $\|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t).\mathbf{e}_2(t) = 0$
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Nothing special about $(1, 0)$ and $(0, 1)$,

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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$
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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$ $\{\mathbf{T}(t), \}$

$\|\mathbf{e}_1(t)\| = 1$, $\|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t).\mathbf{e}_2(t) = 0$

"$\mathbf{e}_1(t)$ and $\mathbf{e}_2(t)$ form an orthonormal basis"

For any, $\mathbf{v}(t) \in \mathbb{R}^2$,

$\mathbf{v}(t) = \alpha(t)\mathbf{e}_1(t) + \beta(t)\mathbf{e}_2(t)$

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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$

$\{\mathbf{T}(t), \mathbf{N}_s(t)\}$

$\|\mathbf{e}_1(t)\| = 1$, $\|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t).\mathbf{e}_2(t) = 0$

"$\mathbf{e}_1(t)$ and $\mathbf{e}_2(t)$ form an orthonormal basis"

For any, $\mathbf{v}(t) \in \mathbb{R}^2$,

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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$
$\|\mathbf{e}_1(t)\| = 1$, $\|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t).\mathbf{e}_2(t) = 0$
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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$

$\|\mathbf{e}_1(t)\| = 1$, $\|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t).\mathbf{e}_2(t) = 0$

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$\{\mathbf{T}(t), \mathbf{N}_s(t)\}$ form an orthonormal basis, for each t.

For any, $\mathbf{v}(t) \in \mathbb{R}^2$,

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for some $\alpha(t), \beta(t) \in \mathbb{R}$ (uniquely represented like this!)

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So, $\mathbf{v}(t), \mathbf{e}_1(t), \mathbf{e}_2(t)$ smooth $\implies$ $\alpha(t)$, $\beta(t)$ smooth.

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Exercise. If γ

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Solution. Let the curvature be κ.

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Exercise. If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization with constant (non-zero) curvature, then show that $\gamma(t)$ lies on a circle for every t.

Solution. Let the curvature be κ.
If it were true, then the (as yet, unknown) center p, would satisfy,

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