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Solution. $\tilde\gamma(\widetilde{t}) = \gamma(s^{-1}(\tilde t))$

$$\begin{aligned}\dot{\tilde\gamma}(\widetilde{t}) &= \dot\gamma(s^{-1}(\tilde t))(s^{-1}(\tilde t))' \\ &= \dot\gamma(s^{-1}(\tilde t))\times ??\end{aligned}$$

□

Definition.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ a regular parametrization.
$\mathbf{T}(t) = \frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}$ is the *unit tangent vector* at t.

Exercise.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ a regular parametrization.
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ a unit speed reparametrization.
Show that,
$\dot{\tilde{\gamma}}(\tilde{t}) = \mathbf{T}(s^{-1}(\tilde{t}))$

Solution. $\tilde{\gamma}(\widetilde{t}) = \gamma(s^{-1}(\tilde{t}))$

$$\begin{aligned} \dot{\tilde{\gamma}}(\widetilde{t}) &= \dot{\gamma}(s^{-1}(\tilde{t}))(s^{-1}(\tilde{t}))' \\ &= \dot{\gamma}(s^{-1}(\tilde{t}))\frac{1}{\|\dot{\gamma}(s^{-1}(\tilde{t}))\|} \end{aligned}$$

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Recall,
If $g(f(t)) = t$, then
$g'(f(t))f'(t) = 1$, therefore, if $f'(t) \neq 0$ for any t,

$$g'(f(t)) = \frac{1}{f'(t)}$$

Taking $f(t) = s(t)$ and $g(t) = s^{-1}(t)$

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Exercise. Show that the curvature at any point of any line

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Exercise. Show that the curvature at any point of any line is 0.

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