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Regular parametrization

Definition.
The point $\gamma(t)$

Regular parametrization

Definition.
The point $\gamma(t)$ of γ :

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to$

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Definition.
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Proposition. *A reparametrization of a regular parametrization*

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$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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But, $\gamma'(t) \neq 0$ □

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□

Inner product:

Inner product:

$v = (2, 3)$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

$v.w$

Inner product:

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$w = (2, 1)$

$v.w := (2, 3).(2, 1)$

Inner product:

$v = (2, 3)$

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$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1$

Inner product:
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$v = (x, y)$
$v.w = ||v|| ||w|| \cos(\theta)$ where,
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