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Rule	**Example**
$(cf)' = cf'$, where $c \in \mathbb{R}$	$(2\sin(x))' = 2\cos(x)$
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$(fg)' = f'g + fg'$	$(x^2\sin(x))' =$

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$(fg)' = f'g + fg'$	$(x^2\sin(x))' =$ $2x\sin(x) + x^2\cos(x)$
$(f/g)' = \frac{f'g - fg'}{g^2}$	$(\frac{\sin(x)}{x^2})' =$ $\frac{\cos(x)x^2 - \sin(x)2x}{x^4}$
$(f(g(x)))' = f'(g(x))g'(x)$	$(\sin(x^2))' =$

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2. $(x^n)' = nx^{n-1}$
3. $(\sin(x))' = \cos(x)$
4. $(\cos(x))' = -\sin(x)$
5. $(\tan(x))' = \sec^2(x)$
6. $(\log(x))' = \frac{1}{x}$
7. $(e^x)' = e^x$

Example.

$$(x^2 \sin(x^3) + \cos(x))' =$$

Rule	Example
$(cf)' = cf'$, where $c \in \mathbb{R}$	$(2\sin(x))' = 2\cos(x)$
$(f+g)' = f' + g'$	$(\sin(x) + x^3)' = \cos(x) + 3x^2$
$(f-g)' = f' - g'$	$(\sin(x) - x^3)' = \cos(x) - 3x^2$
$(fg)' = f'g + fg'$	$(x^2\sin(x))' = 2x\sin(x) + x^2\cos(x)$
$(f/g)' = \frac{f'g - fg'}{g^2}$	$(\frac{\sin(x)}{x^2})' = \frac{\cos(x)x^2 - \sin(x)2x}{x^4}$
$(f(g(x)))' = f'(g(x))g'(x)$	$(\sin(x^2))' = \cos(x^2)2x$

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Example

Example

$\gamma :$

Example

$\gamma : (-\pi,$

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$\gamma : (-\pi, \pi) \to \mathbb{R}^2$

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$$\sqrt{v_1(t)^2 + v_2(t)^2}$$

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Can we find a parametrization of the circle

Example

Making the "speed" 1

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Can we find a parametrization of the circle to ensure the speed is 1?

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$\gamma : (-r\pi, r\pi) \to \mathbb{R}^2$
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