Exercise sheet 4

Curves and Surfaces, MTH201

1. Prove that $\ddot{\gamma}(t).\hat{\mathbf{n}}(\gamma(t)) = -\dot{\gamma}(t).\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{n}}(\gamma(t))$. Here $\hat{\ltimes}$ denotes the normal to a surface. (*Hint*: You have done similar things many times before. Which rule helps you?)

2. Use the chain rule to show that $\frac{\mathrm{d}}{\mathrm{d}t}|_{t=t_0}\hat{\mathbf{n}}(\gamma(t)) = \frac{\mathrm{d}}{\mathrm{d}t}|_{t=t_0}\hat{\mathbf{n}}(\delta(t))$ if $\gamma(t_0) = \delta(t_0)$ and $\dot{\gamma}(t_0) = \dot{\delta}(t_0)$. In other words, the derivative is the same for curves which pass through the same point $\gamma(t_0)$ and have the same velocity vectors. (*Hint*: Look at everything from the point of view of the coordinate patch and apply chain rule).

3. Can you see how the previous two exercises show that for a unit speed parametrization γ, the quantity $\ddot{\gamma}(t).\hat{\mathbf{n}}(\gamma(t))$ depends only on the point and direction.

4. Prove that $\hat{\mathbf{n}}(\gamma(t))$ is perpendicular to $\mathbf{T}(t)$, where $\mathbf{T}(t)$ is the unit tangent of γ at t.

5. Consider a parametrization, $\gamma(t)$ and let $\mathbf{N}(t)$ denote its unit normal at t. Prove that $\mathbf{N}(t) = \hat{\mathbf{n}}(\gamma(t))$ if and only if $\kappa_g(t) = 0$

6. Consider the sphere of radius 1. Give it a surface patch and use that to compute the normal curvature of a unit speed parametrization of any curve on the sphere at any given point.

7. Consider any plane. Give it a surface patch and use that to compute the normal curvature of a unit speed parametrization of any curve on that plane at any given point.