Exercise sheet 7

Curves and Surfaces, MTH201

For all these questions, $\hat{\mathbf{n}}(p) := \sigma_x(p) \times \sigma_y(p)$

1. Prove that $\hat{\mathbf{n}}(\gamma(t))$ is perpendicular to $\mathbf{T}(t)$.
2. Prove that $\mathbf{N}(t) = \hat{\mathbf{n}}(t)$ if and only if $\kappa_g(t) = 0$
3. Prove that the area can be expressed entirely in terms of the first fundamental form.

$$A_\sigma(R) = \int_R \sqrt{E(x,y)G(x,y) - F^2(x,y)}\mathrm{d}x\mathrm{d}y$$

4. How does the matrix of the first fundamental form vary with a coordinate transformation?
5. Prove that if the surface patch is regular then the matrix of the first fundamental form is invertible.
6. Recall the definition of $D_p(f) : T_p(S) \to T_p(S)$
   (a) Show that $D_p(Id_S) = Id_{T_p(S)}$
   (b) Show that $D_p(f \circ g) = D_p(f) \circ D_p(g)$ where $g : S_1 \to S_2$ and $f : S_2 \to S_3$ are smooth functions between surfaces.
   (c) Prove that if f is smooth with a smooth inverse, then $D_p(f)$ is invertible.
7. Prove that $\mathcal{W}\mathbf{v}.\mathbf{w} = \mathcal{W}\mathbf{w}.\mathbf{v}$