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$$s_\beta(t)$$

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Prove that $s_\beta(t) - s_\alpha(t)$ is a constant.

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Theorem (First Fundamental theorem of calculus)**.**
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Proof.

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Corollary. *The arc length function $s(t)$*

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$$s(t) := \int_{t_0}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$$

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Taking $f(t) = s(t)$ and $g(t) = s^{-1}(t)$

$$(s^{-1})'(s(t)) = \frac{1}{s'(t)}$$

Theorem (First Fundamental theorem of calculus). *$f : [\alpha, \beta] \to \mathbb{R}$ continuous*

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Theorem (First Fundamental theorem of calculus)**.** $f : [\alpha, \beta] \to \mathbb{R}$ *continuous*

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Corollary. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ *is a smooth and regular parametrization.*
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$\gamma : (\alpha, \beta) \rightarrow$

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If $\phi(t) = s^{-1}(t)$,

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$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$

If $\phi(t) = s^{-1}(t)$,
$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$

$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$

Proved that,

Theorem.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ *a regular smooth parametrization*

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ *a regular smooth parametrization*
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$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$

Proved that,

Theorem.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ *a regular smooth parametrization*
$s(t)$, *the arc-length from* t_0 *to* t

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
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$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$

$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$

Proved that,

Theorem.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ a regular smooth parametrization
$s(t)$, the arc-length from t_0 to t (where $t, t_0 \in (\alpha, \beta)$)

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$

If $\phi(t) = s^{-1}(t)$,
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Proved that,

Theorem.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ a regular smooth parametrization
$s(t)$, the arc-length from t_0 to t (where $t, t_0 \in (\alpha, \beta)$)
$\tilde{\gamma}(t) = \gamma(s^{-1}(t))$, then

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$

If $\phi(t) = s^{-1}(t)$,
$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$

$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$

Proved that,

Theorem.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ a regular smooth parametrization
$s(t)$, the arc-length from t_0 to t (where $t, t_0 \in (\alpha, \beta)$)
$\tilde{\gamma}(t) = \gamma(s^{-1}(t))$, then
$\tilde{\gamma}$ is a unit speed re-parametrization.