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Explicitly,
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Examples.

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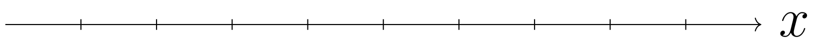
Parametrizing a circle



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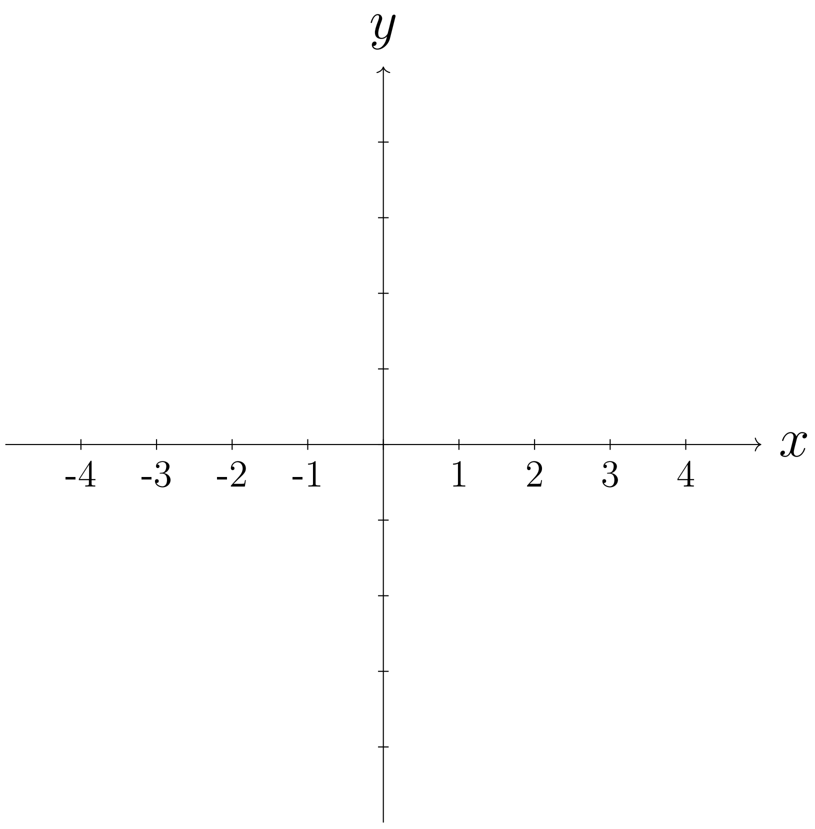

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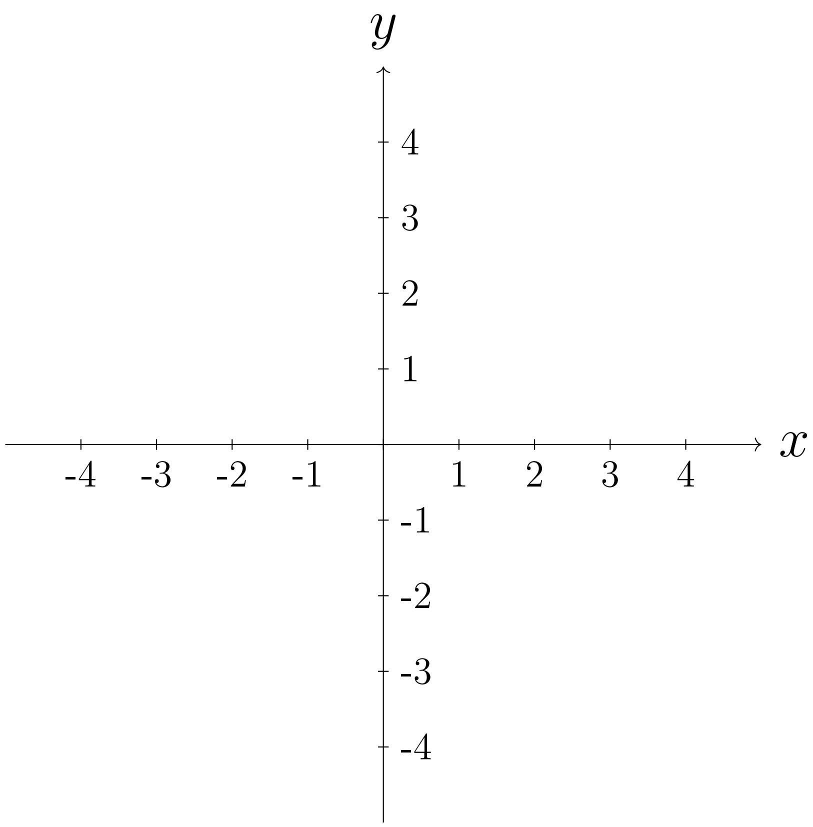

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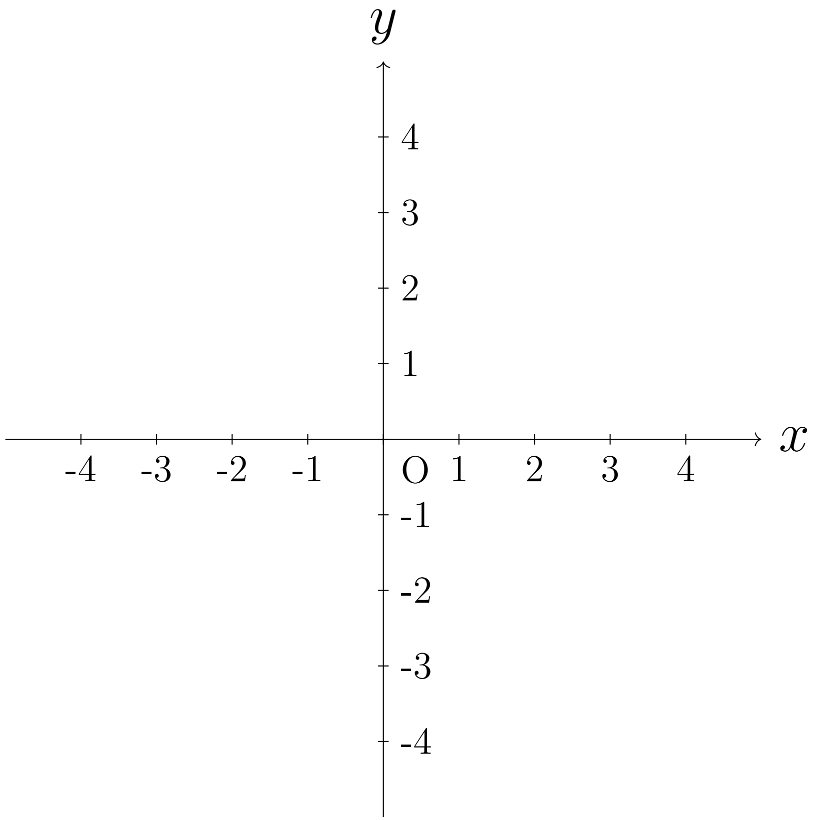
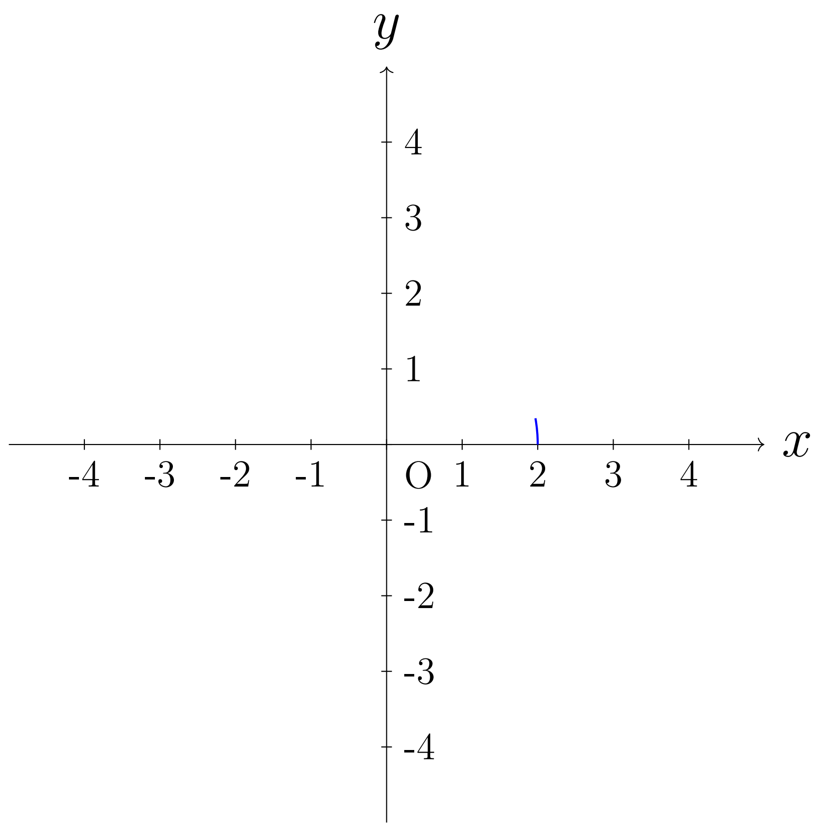


Parametrizing a circle

$\gamma : (0, \pi/18) \to \mathbb{R}^2$



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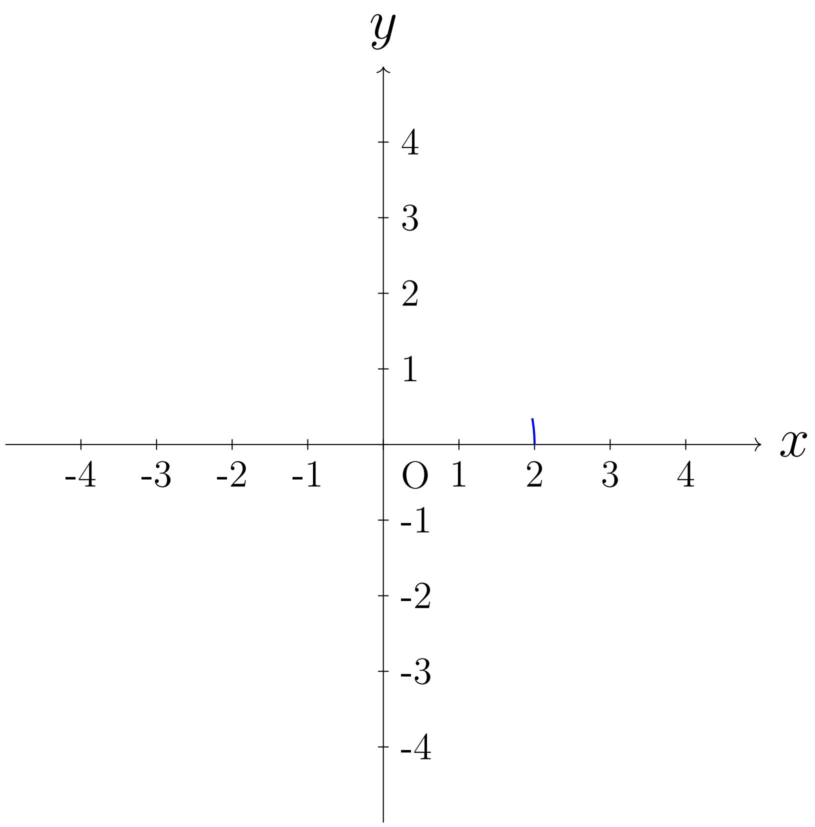
$$\gamma : (0, \pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



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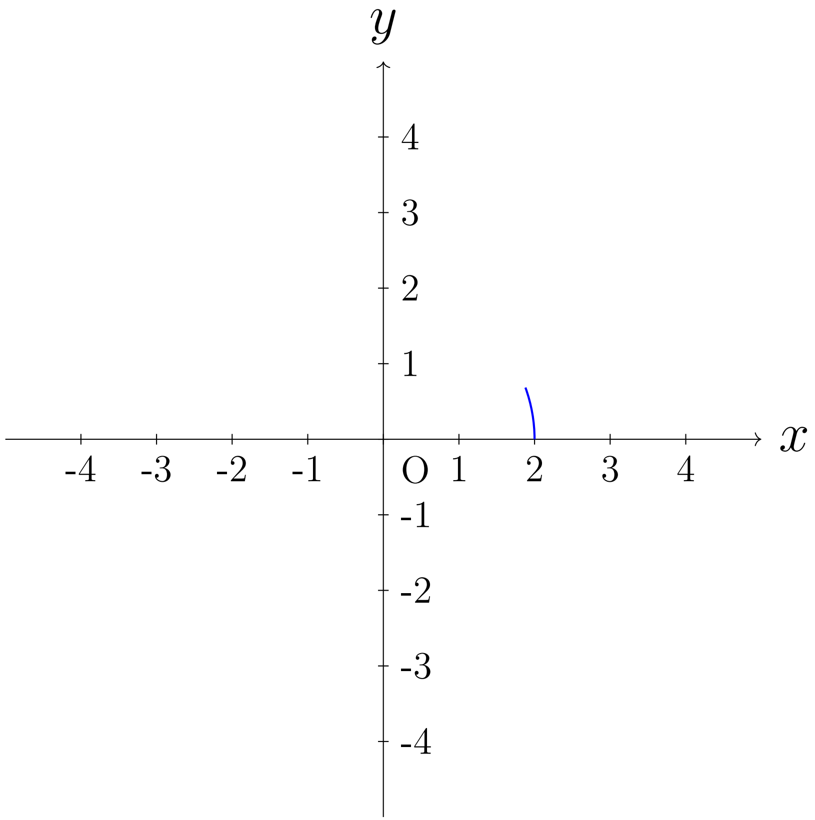
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



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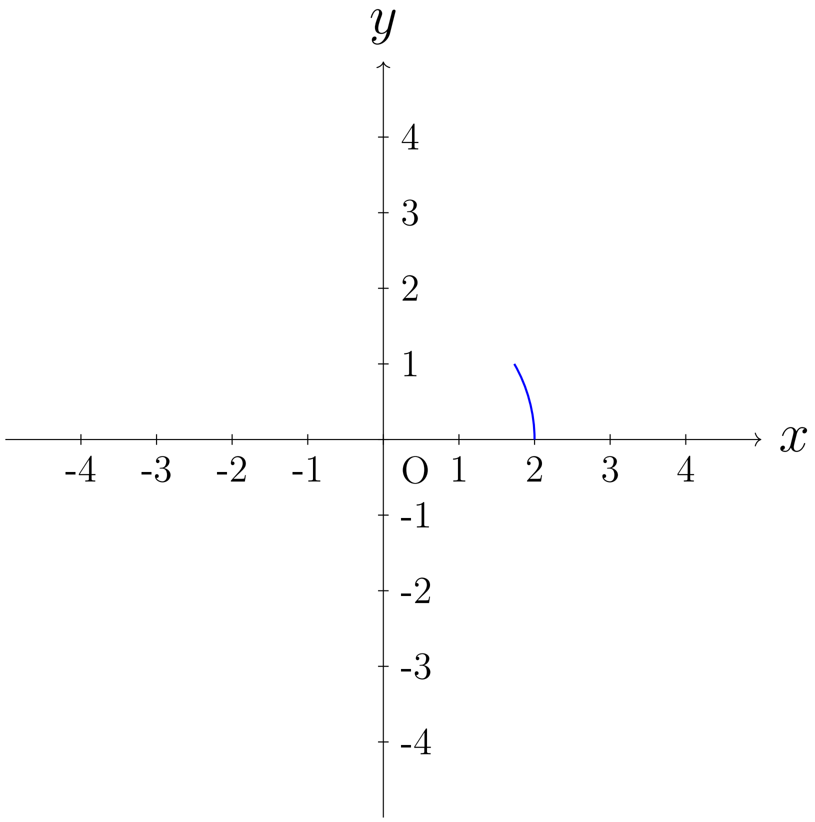
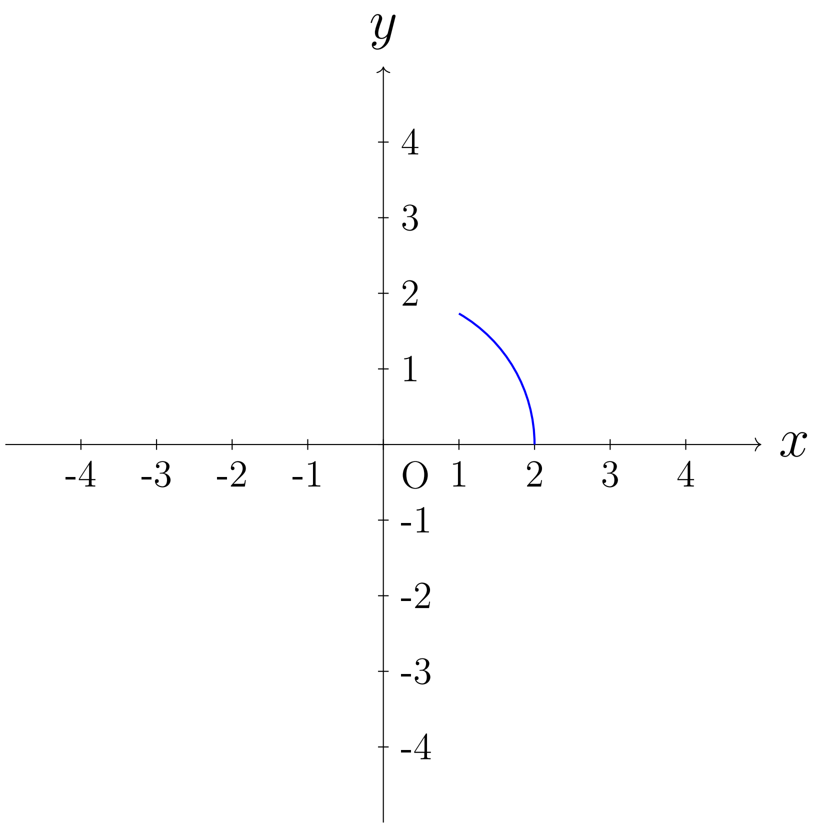
$$\gamma : (0, 3\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



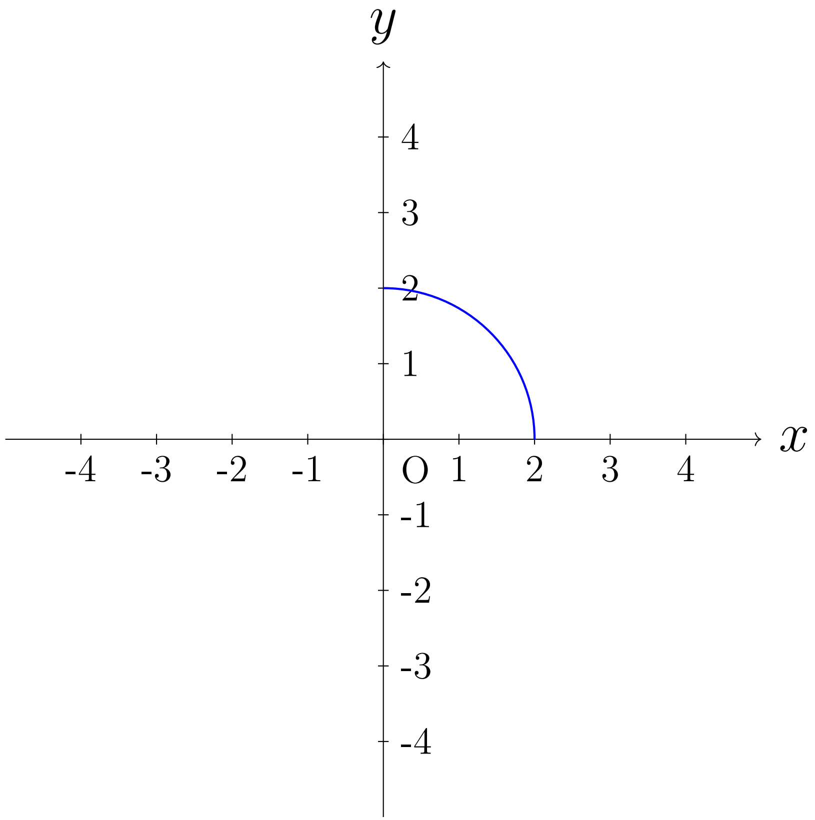
Parametrizing a circle

$$\gamma : (0, 6\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



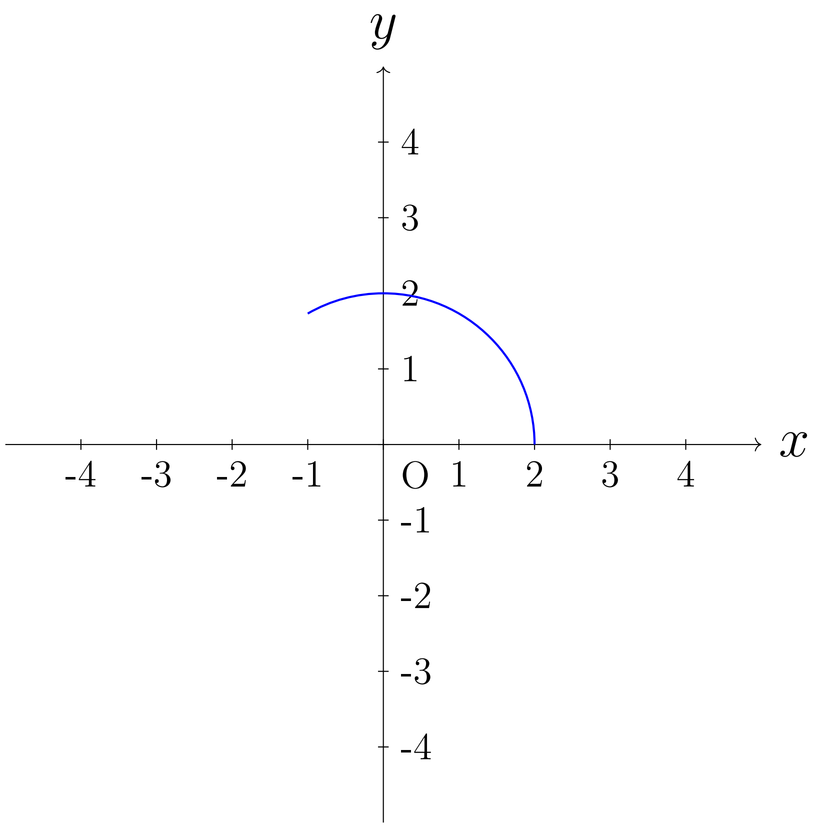
Parametrizing a circle

$$\gamma : (0, 9\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



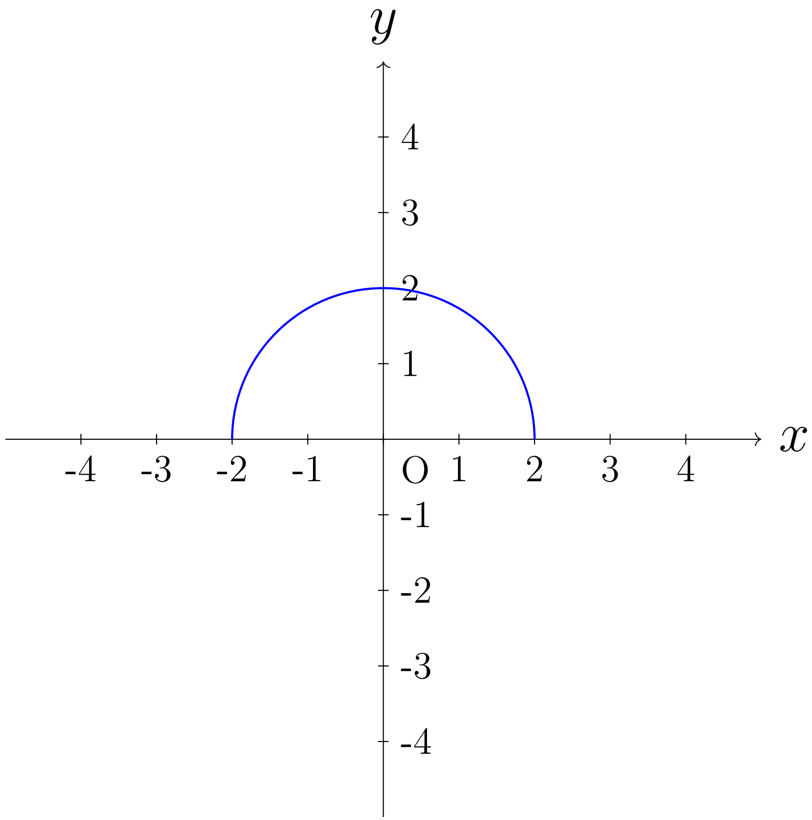
Parametrizing a circle

$$\gamma : (0, 12\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



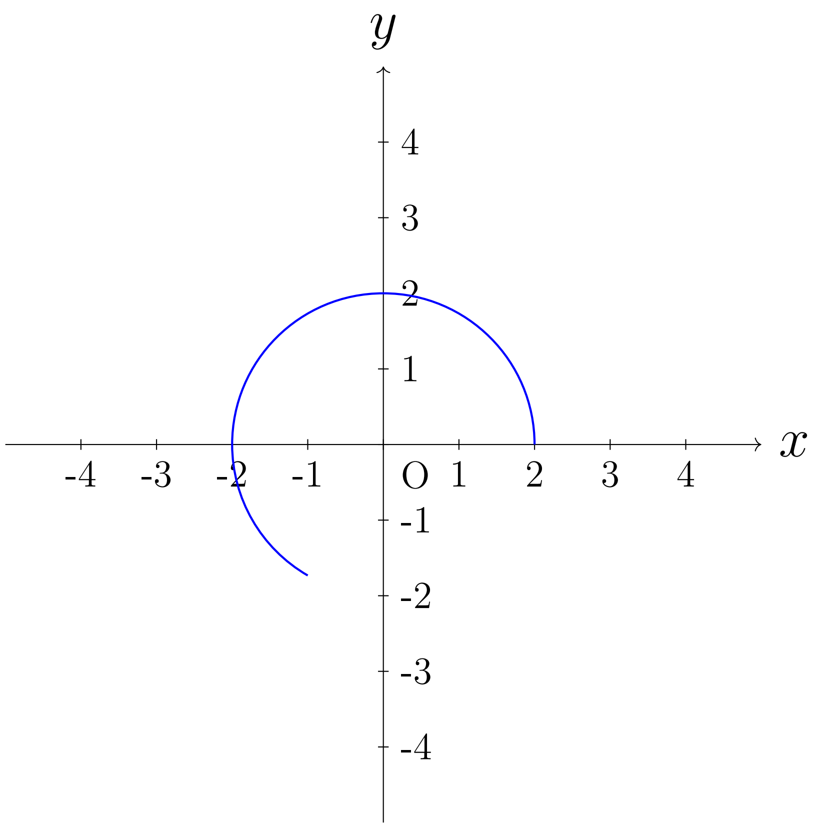
Parametrizing a circle

$$\gamma : (0, 18\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



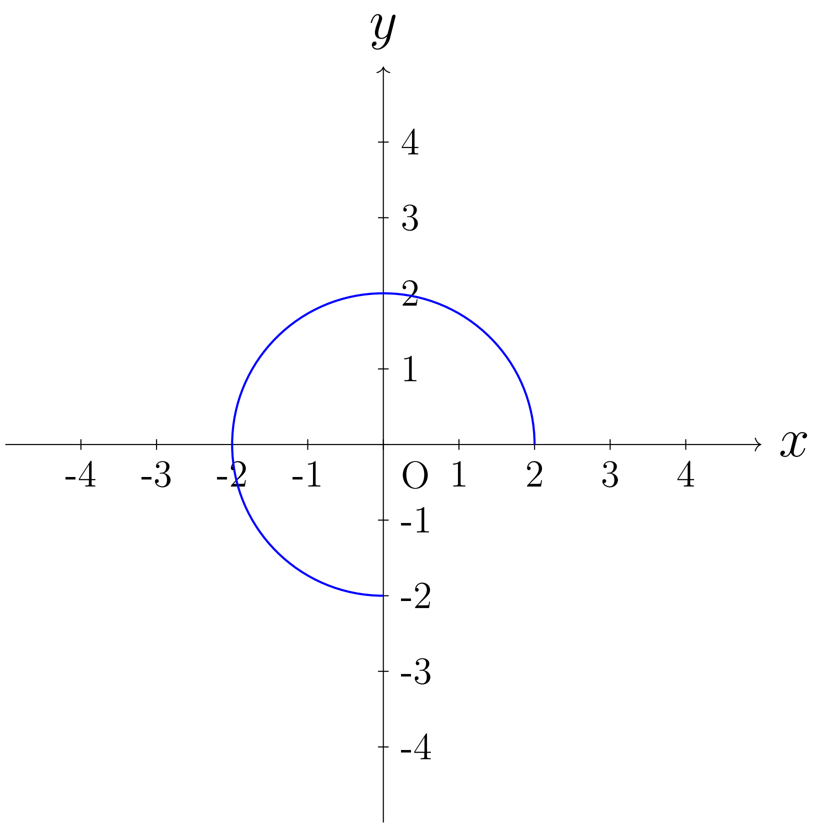
Parametrizing a circle

$$\gamma : (0, 24\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



Parametrizing a circle

$$\gamma : (0, 27\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



Parametrizing a line

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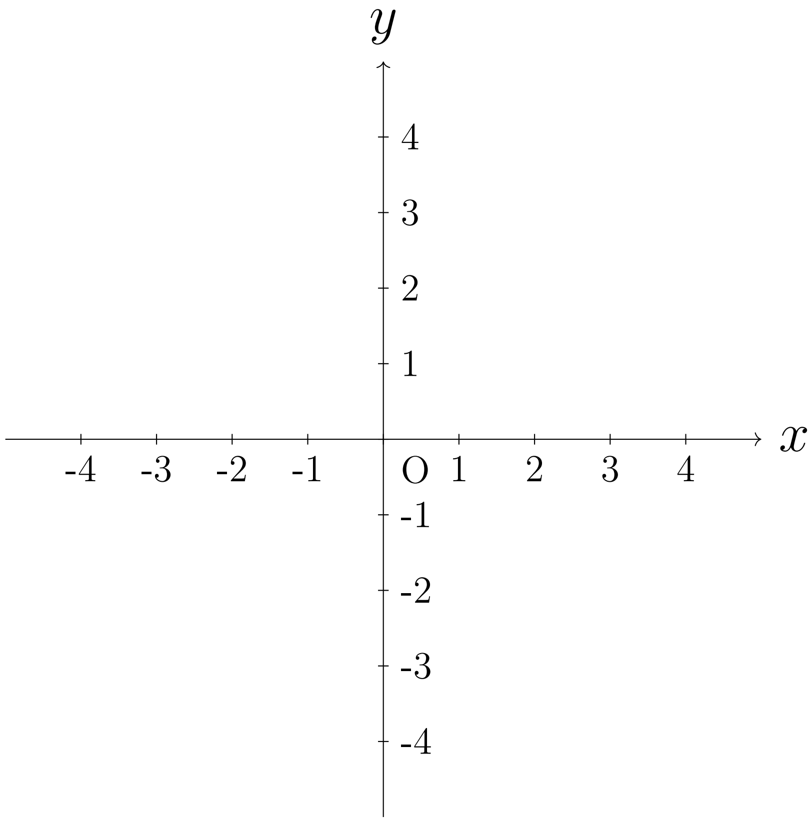
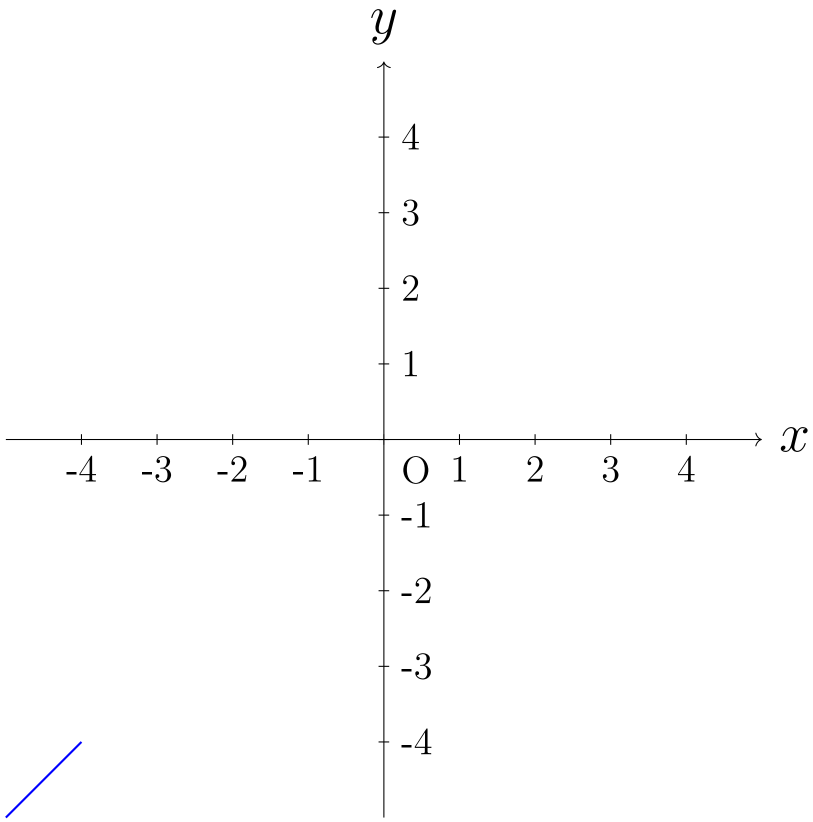


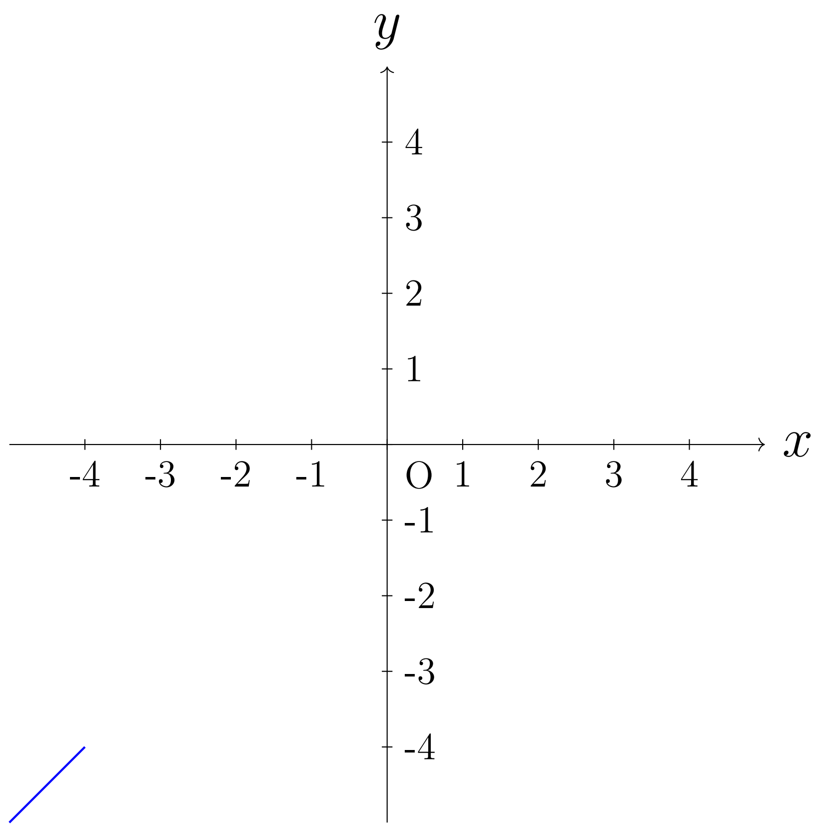
Parametrizing a line

$\gamma : (-5, -4) \to \mathbb{R}^2$



Parametrizing a line

$$\gamma : (-5, -4) \to \mathbb{R}^2$$
$$\gamma(t) := (t, t)$$



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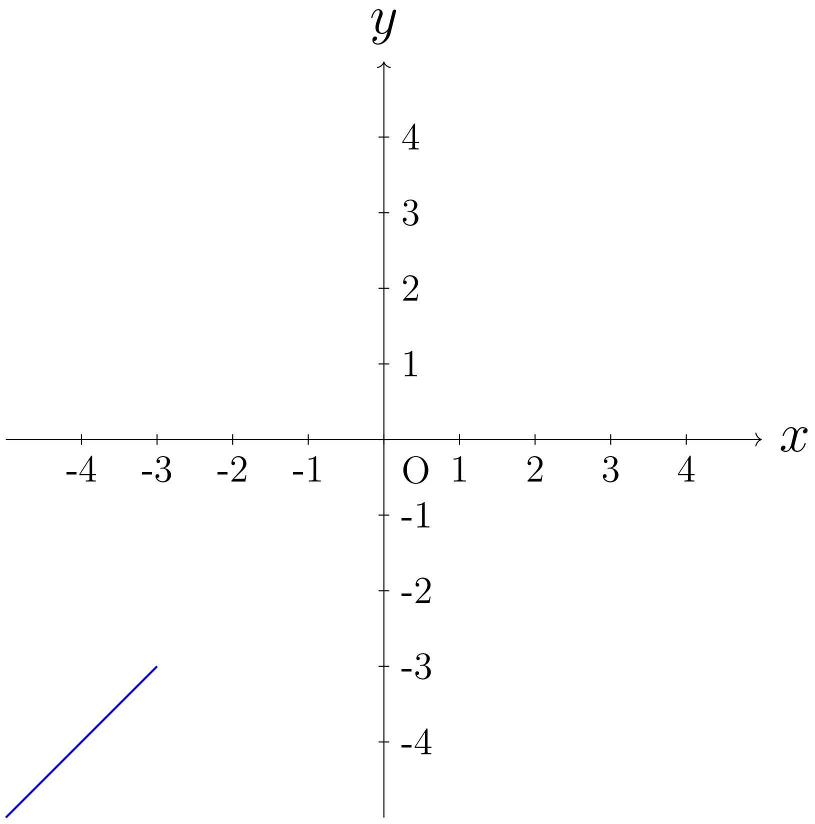

Parametrizing a line

$$\gamma : (-5, -2) \to \mathbb{R}^2$$
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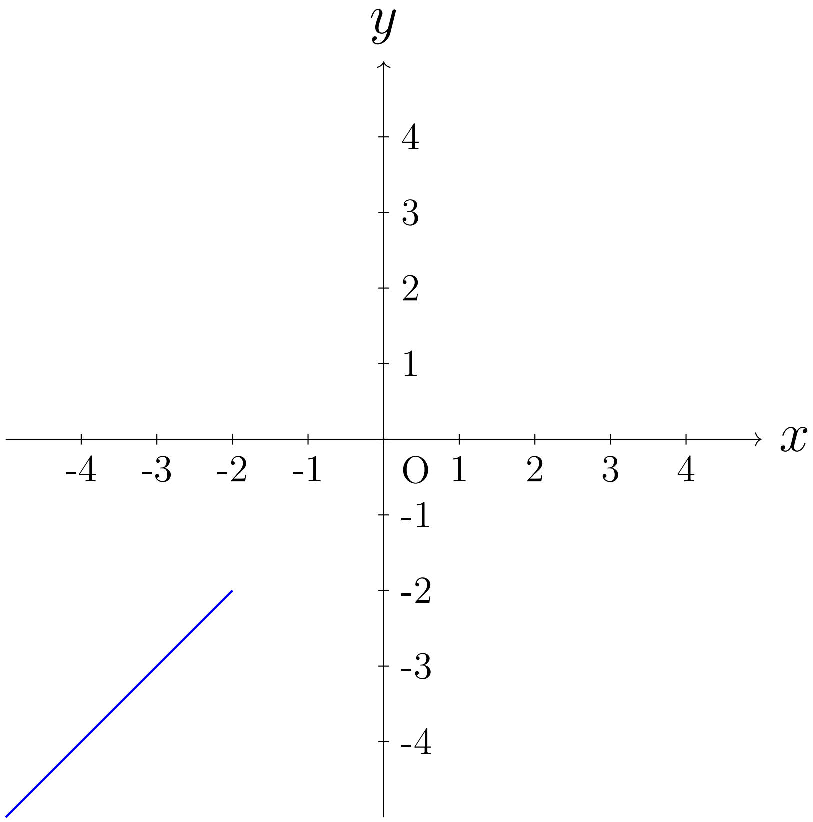

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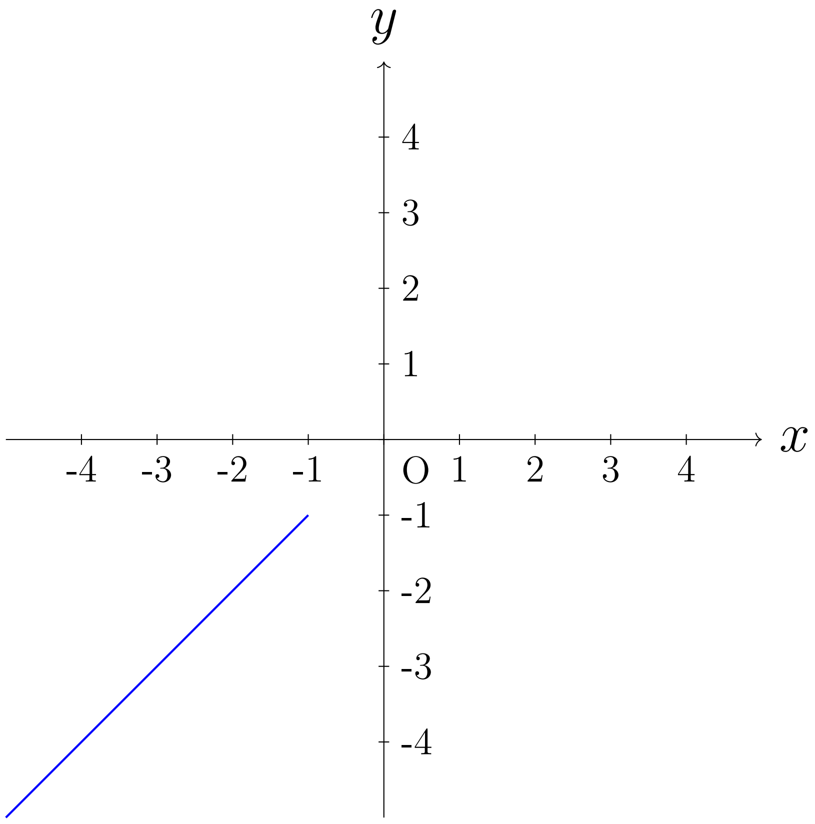

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$$\gamma : (-5, 0) \to \mathbb{R}^2$$
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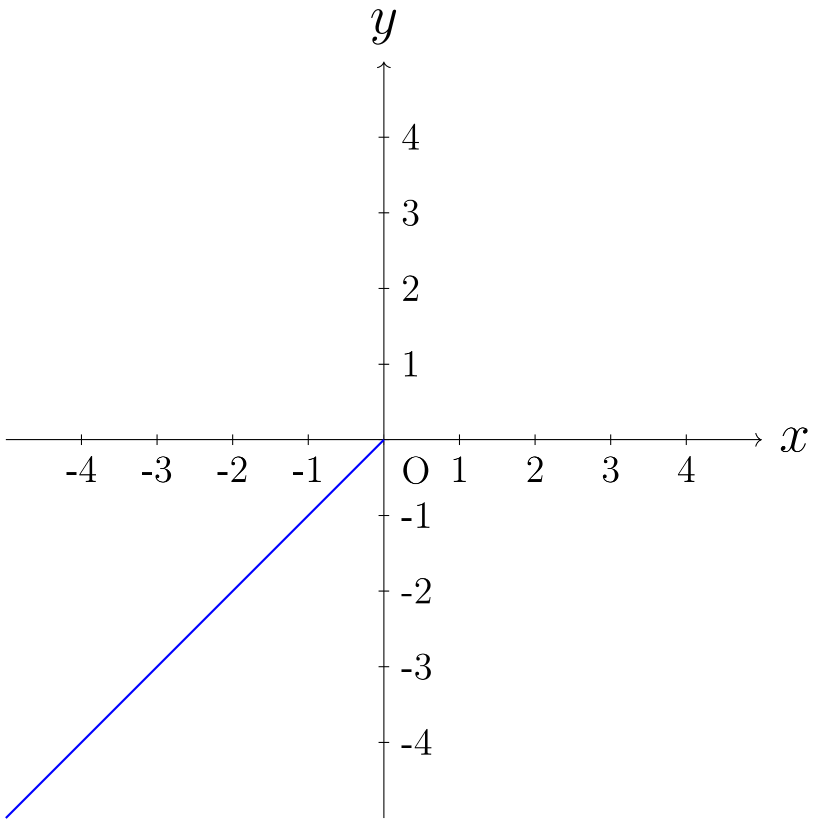

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$$\gamma : (-5, 1) \to \mathbb{R}^2$$
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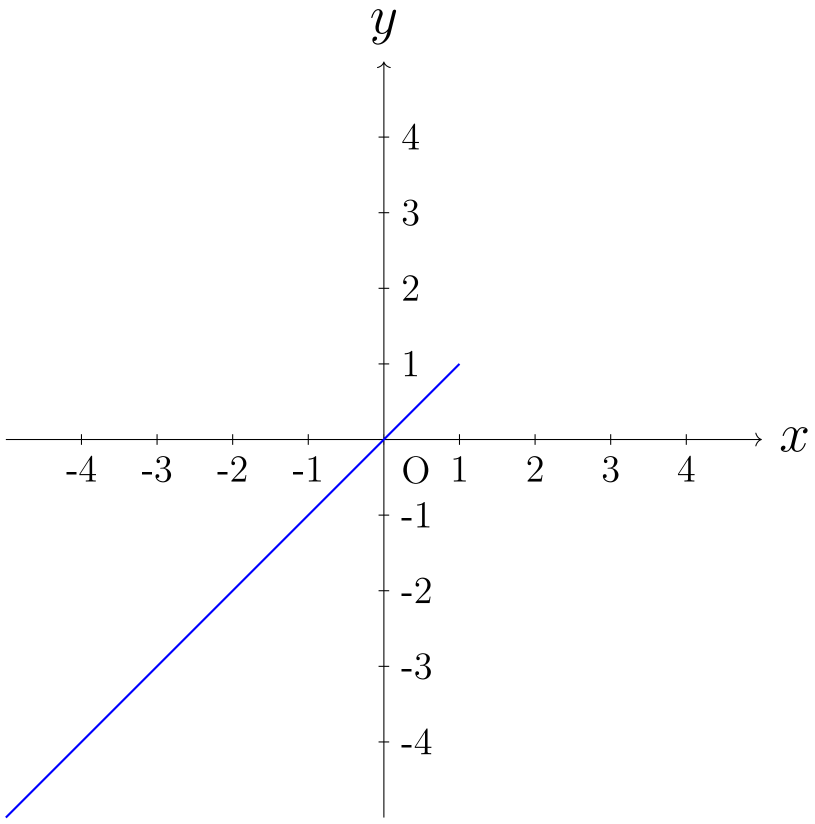

Parametrizing a line

$$\gamma : (-5, 2) \to \mathbb{R}^2$$
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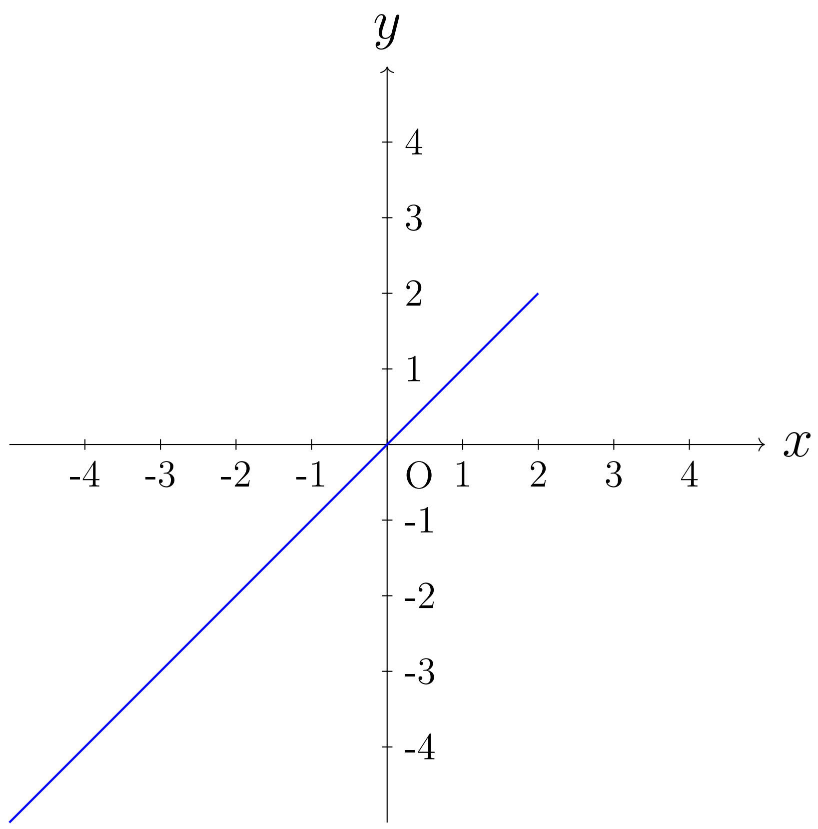

Parametrizing a line

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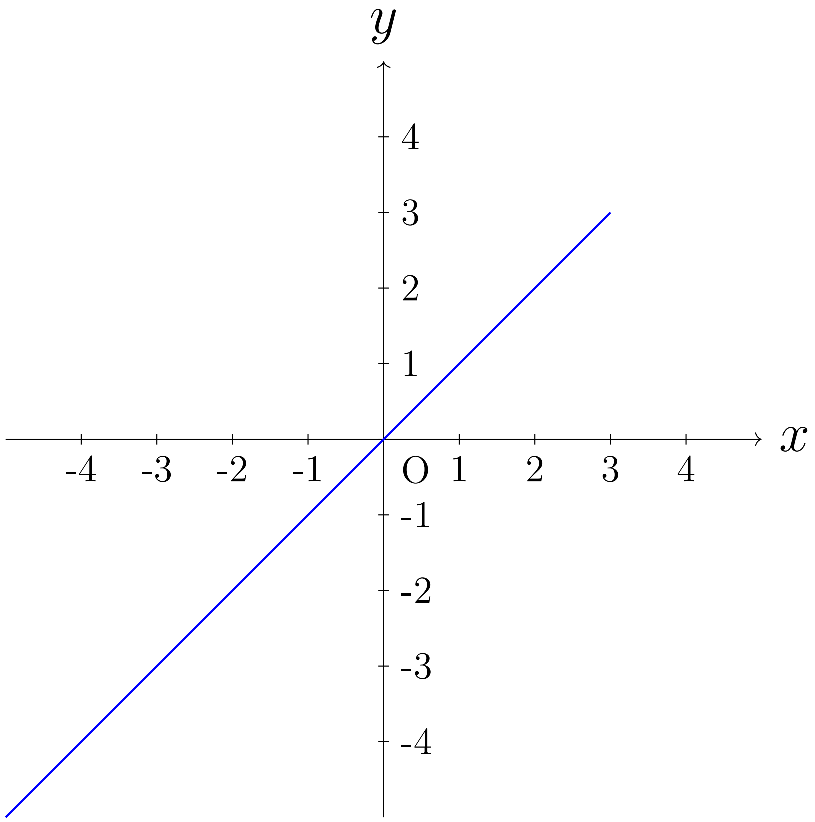

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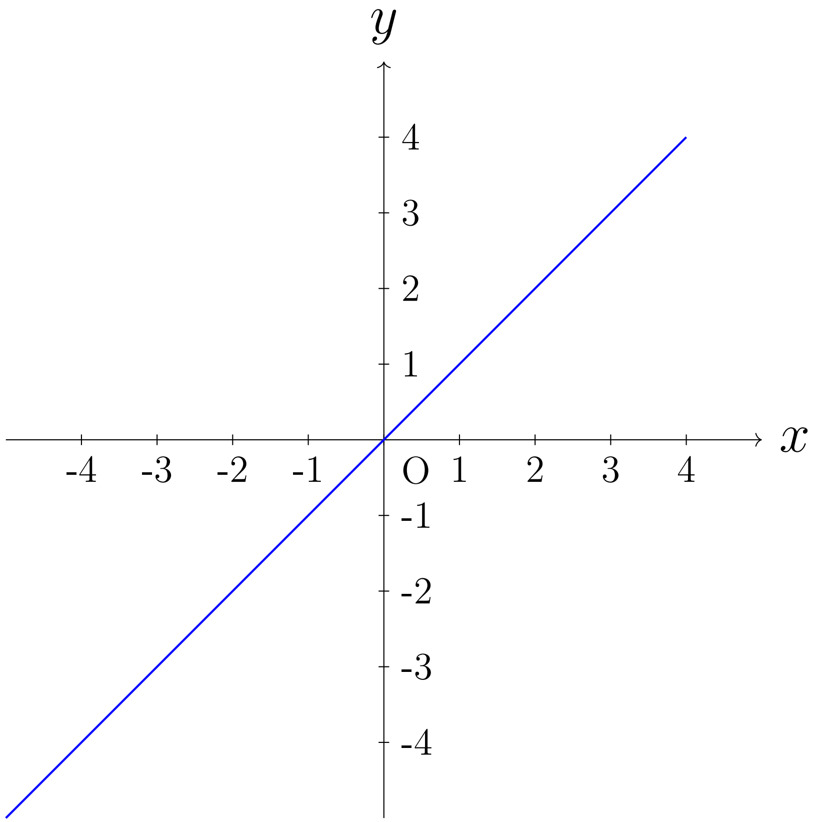

$$\gamma : (-5, 4) \to \mathbb{R}^2$$
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Parametrizing a line

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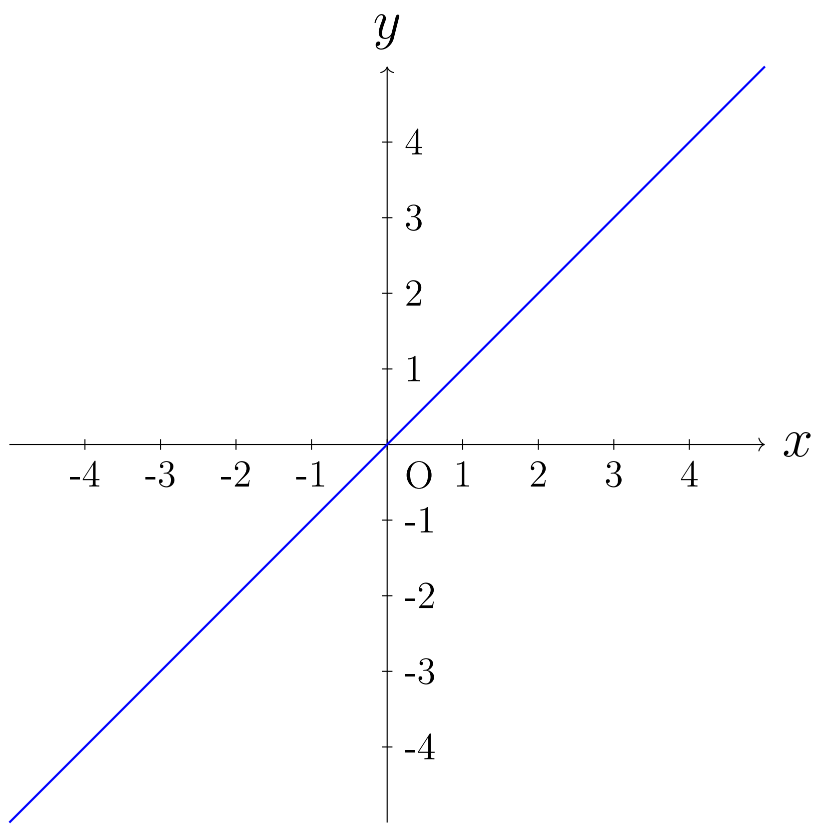

Quick review: Derivative

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Example. $f : \mathbb{R} \to \mathbb{R}$

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$$f(x) = \{$$

Quick review: Derivative

Example. $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 & x < 5 \end{cases}$$

Quick review: Derivative

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$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \end{cases}$$

Quick review: Derivative

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$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \\ x^3 & x > 5 \end{cases}$$

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Example.

$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \\ x^{\color{red}2} & x > 5 \end{cases}$$

Quick review: Derivative

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Example.

$$f(x) = \begin{cases} x^2 & x \neq 5 \\ 0 & x = 5 \end{cases}$$

Quick review: Derivative

Example. $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \\ x^3 & x > 5 \end{cases}$$

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f is "continous".

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